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MHD STEADY LAMINAR GENERALIZED COUETTE FLOW OF CONDUCTING VISCOUS FLUID THROUGH POROUS MEDIUM BETWEEN TWO PARALLEL PLATES

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Abstract

In the present paper, the Magnetohydrodynamic steady laminar generalized couette flow of a conducting viscous fluid through porous medium between two parallel flat plates under the influence of uniform magnetic field applied perpendicularly to the flow of fluid has been studied. There has also been discussed the skin friction, average, maximum and minimum velocities, the volumetric flow and energy losses in the channel. The particular case, when magnetic field and porous medium both are withdrawn, have also been considered.

Nomenclature

u=Velocity component along x-axis, v=Velocity component along y-axis
w=Velocity component along z-axis, t=The time
p=Fluid pressure, ρ=Density of fluid
μ=Coefficient of viscosity, ν=Kinematic viscosity
σ=The electrical conductivity, K=The permeability of porous medium
B_0=The magnetic inductivity , Q=The volumetric flow
C_f=Drag coefficient at y=0, C_f'=Drag coefficient at y=h
u_{avg}=Average velocity, u_{max}=Maximum velocity
u_{min}=Minimum velocity, σ_{xy}=Shearing stress
U=Constant velocity at y=0, h=Vertical distance between two parallel flat plates
$[\sigma_{xy}]_{y=0}$=Coefficient of skin friction at y=0, $[\sigma_{xy}]_{y=h}$= Coefficient of skin friction at y=h

Introduction

Gupta and Jain[1] investigated the couette flow of an electrically conducting viscous fluid. Helmy[2] has studied on unsteady MHD flow of viscous conducting fluid. MHD flows with different situation have been discussed by many researchers such as: Sengupta and Kumar[6]; Kumar[4]; Sharma, Singh, and Chandramouli[7];Varshney and Kumar[8]; Radhakrishnamacharya and Rao[5] ; Kumar and Singh[3]; Tripathi, Sharma and Singh[9]; Kima, Manyonge, Bitok, Adenyah and Barasar[10]; Fazurddin, Sreekanath, and Raju[11]; Mollah, Islam, Khatun, and Alam[12] etc.

The aim of the present paper is to study the steady laminar generalized couette flow of a conducting viscous fluid through porous medium between two parallel flat plates under the influence of transverse uniform magnetic field. There have also been discussed the skin friction, average, maximum and minimum velocities, and the volumetric flow and energy losses in the channel in detail. Particular case, when magnetic field and porous medium are withdrawn, have also been considered.

Formulation Of The Problem

Considering the steady laminar generalized plane couette flow of viscous incompressible electrically conducting fluid through porous medium between two parallel flat plates AB and CD such that $y=0$ and $y=h$ respectively of infinite length in x and z-direction in the presence of uniform transverse magnetic field B_0 applied parallel to y-axis. Initially (i.e. when time $t\leq 0$) fluid and plates of the channel are assumed to be at rest. When $t>0$ the lower plate ($y=0$) is kept fixed and upper plate ($y=h$) starts moving with constant velocity U.

Since magnetic Reynolds number is very small for metallic liquids and partially ionized fluids so the induced magnetic field can be neglected.

Under the above assumptions, the Navier-Stokes equations of motion for viscous incompressible electrically conducting fluid through porous medium in the absence of body force are

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{v}{K}w-\frac{\sigma B_0^2 u}{\rho} \quad \text{...(1)}$$

and $$0=-\frac{\partial p}{\partial y} \quad \text{...(2)}$$

From equation (2) $\frac{\partial p}{\partial y}=0 \Rightarrow p$ is independent of y.

So p is the function of x only which may be written as

$$p = p(x) \qquad \dots(3)$$

The initial and boundary conditions are

$$\left.\begin{array}{lll} u = 0, & 0 \le y \le h & \text{for } t \le 0 \\ u = 0, & \text{at } y = 0 & \text{for } t > 0 \\ u = U, & \text{at } y = h & \text{for } t > 0 \end{array}\right\} \qquad \dots(4)$$

Introducing the following non-dimensional quantities

$$x^{*} = \frac{x}{h}\ ,\ y^{*} = \frac{y}{h}\ ,\ u^{*} = \frac{h}{\nu}u\ ,\ t^{*} = \frac{\nu}{h^2}t\ ,\ p^{*} = \frac{h^2}{\rho \upsilon^2}p$$

in equation (1), it is found (after dropping the stars)

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{K} + H^2\right)u \qquad \dots(5)$$

where $H = B_0 h\sqrt{\sigma/\mu}$ (Hartmann number)

The initial and boundary conditions becomes

$$\left.\begin{array}{lll} u = 0, & 0 \le y \le 1 & \text{for } t \le 0 \\ u = 0, & \text{at } y = 0 & \text{for } t > 0 \\ u = U_0, & \text{at } y = 1 & \text{for } t > 0 \end{array}\right\} \qquad \dots(6)$$

Solution Of The Problem

From equation (5)

$$\left[\frac{\partial^2}{\partial y^2} - \left(\frac{1}{K} + H^2\right)\right]u = \frac{\partial p}{\partial x} = P_0 \qquad \dots(7)$$

where $\frac{\partial p}{\partial x}$ = constant = P_0 (say)

From above equation, we have

$$u(y) = C_1 e^{Hy} + C_2 e^{-Hy} - \frac{P_0}{\frac{1}{K} + H^2} \qquad \dots(8)$$

Applying boundary conditions given by equation (6), there is found velocity of viscous fluid

$$u(y) = \frac{\frac{P_0}{\frac{1}{K}+H^2}(1-e^{-H}) + U_0}{e^{H} - e^{-H}} e^{Hy} + \frac{\frac{P_0}{\frac{1}{K}+H^2}(e^{H}-1) - U_0}{e^{H} - e^{-H}} e^{-Hy} - \frac{P_0}{\frac{1}{K}+H^2} \qquad \dots(9)$$

where $C_1 = \frac{\frac{P_0}{\frac{1}{K}+H^2}(1-e^{-H}) + U_0}{e^{H} - e^{-H}}$, $C_2 = \frac{\frac{P_0}{\frac{1}{K}+H^2}(e^{H}-1) - U_0}{e^{H} - e^{-H}}$ ...(10)

Average Vilocity

The average velocity is given by

$$u_{avg} = \int_0^1 u\, dy$$

$$= \int_0^1 \left\{ C_1 e^{Hy} + C_2 e^{-Hy} - \frac{P_0}{\frac{1}{K} + H^2} \right\} dy$$

$$= \left[\frac{C_1}{H} e^{Hy} - \frac{C_2}{H} e^{-Hy} - \frac{P_0}{\frac{1}{K}+H^2} y \right]_0^1$$

$$= \frac{C_1}{H}(e^H - 1) + \frac{C_2}{H}(1 - e^{-H}) - \frac{P_0}{\frac{1}{K}+H^2} \quad \ldots (11)$$

The Volumetric Flow Per Unit Time

$$Q = u_{avg}$$

$$= \frac{C_1}{H}(e^H - 1) + \frac{C_2}{H}(1 - e^{-H}) - \frac{P_0}{\frac{1}{K} + H^2} \quad \ldots (12)$$

Maximum And Minimum Velocities

By necessary condition for max. and min.

$$\frac{du}{dy} = 0$$

$$\Rightarrow H(C_1 e^{Hy} - C_2 e^{-Hy}) = 0$$

$$\Rightarrow e^{2Hy} = \frac{C_2}{C_1}$$

$$\Rightarrow y = \frac{1}{2H} log_e \left(\frac{C_2}{C_1} \right)$$

Now $$\frac{d^2u}{dy^2} = \left(\frac{1}{K} + H^2 \right)(C_1 e^{Hy} + C_2 e^{-Hy}) \quad \ldots (13)$$

Maximum velocity if $\frac{d^2u}{dy^2} = -\text{ve}$

$$\Rightarrow \left(\frac{1}{K} + H^2 \right)(C_1 e^{Hy} + C_2 e^{-Hy}) < 0$$

$$\Rightarrow C_1 e^{Hy} + C_2 e^{-Hy} < 0$$

$$\Rightarrow e^{2Hy} < -\frac{C_2}{C_1}$$

$$\Rightarrow y < \frac{1}{2H} log_e\left(-\frac{C_2}{C_1}\right) \qquad \ldots(14)$$

Minimum velocity if $\frac{d^2u}{dy^2} = +\text{ve}$

$$\Rightarrow y > \frac{1}{2H} log\left(-\frac{C_2}{C_1}\right) \qquad \ldots(15)$$

Shearing Stress

$$\sigma_{xy} = \mu\frac{du}{dy} = \mu H(C_1e^{Hy} - C_2e^{-Hy}) \qquad \ldots(16)$$

Skin Friction

At lower plate

$$[\sigma_{xy}]_{y=0} = \mu H(C_1 - C_2) \qquad \ldots(17)$$

At upper plate

$$[\sigma_{xy}]_{y=1} = \mu H(C_1e^{H} - C_2e^{-H}) \qquad \ldots(18)$$

Energy Losses In Channels Or Coefficient Of Frictions

$$C_f = \frac{[\sigma_{xy}]_{y=0}}{\frac{1}{2}\rho u_{avg}^2} = \frac{2\mu H(C_1 - C_2)\left(\frac{1}{K} + H^2\right)}{\rho\left[C_1(e^H - 1) + C_2(1 - e^{-H}) - \frac{P_0}{\frac{1}{K} + H^2}\right]^2} \qquad \ldots(19)$$

$$C_f' = \frac{[\sigma_{xy}]_{y=1}}{\frac{1}{2}\rho u_{avg}^2} = \frac{2\mu H(C_1e^H - C_2e^{-H})\left(\frac{1}{K} + H^2\right)}{\rho\left[C_1(e^H - 1) + C_2(1 - e^{-H}) - \frac{P_0}{\frac{1}{K} + H^2}\right]^2} \qquad \ldots(20)$$

Particular Case

1. If magnetic field is withdrawn, all the above results can be obtained by putting H=0 in equations (9)-(20).
2. If porous medium is withdrawn all the above results can be obtained by putting K=∞ in equations (9)-(20).

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